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APPLICATIONS OF INVERSE RELIABILITY IN EARTHQUAKE ENGINEERING DESIGN PROBLEMS WITH MULTIPLE SOLUTIONS

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ABSTRACT

A modified inverse reliability method is proposed in this paper for the case when multiple design solutions exist for the same specified reliability level. An application to the design of a single pile foundation under earthquake loading is presented to demonstrate the method's applicability. The influence of the number of random variables considered in the analysis is also discussed.

INTRODUCTION

The main objective in the design of an engineering system is to ensure that it fulfills its function satisfactorily and safely during its service life, a period during which it would be subjected either to extreme or to service level load demands. The performance of a designed system involves the interaction of many intervening random variables plus those *design parameters* which are chosen by the designer on the basis of safety and serviceability criteria. Given the uncertainty associated with the intervening variables, the performance criteria can be satisfied only with a certain probability of non-performance. The objective of reliability-based design is to determine the design parameters so that the risk of non-performance does not exceed a specified target level.

When the design parameters are given, the reliability of the system can be assessed. This is a *forward reliability* evaluation problem. On the other hand, when the parameters are to be determined on the basis of specified reliability levels, the problem becomes one of *inverse reliability*, which thus defines the nature of reliability-based design.

During the past few years, the inverse reliability method has been discussed both in the context of a single design parameter (Der Kiureghian et al., 1994) or multiple parameters (Li and Foschi, 1998). The method has been shown, through several applications, to be superior to the more traditional approach of trial-and-error, in which the forward reliability method is repeatedly applied until the design parameters correspond to the reliability desired. However, in applications of earthquake engineering, the method could encounter numerical difficulties due to the possible existence of multiple solutions. This normally would occur due to resonant behaviour, when the relationship between maximum structural response and the system properties presents local extrema, as shown in Figure 1.

In another paper (Foschi and Li, 1999) the authors presented the problem of a single pile foundation subjected to earthquake excitation. The objective was the determination of the mass M which the pile can carry so that a serviceability criterion associated with the maximum lateral displacement at the pile cap is satisfied. It was shown in that paper that, in a first approximation, all intervening random variables can be taken as deterministic and at their mean value, while allowing only the most important variable to remain random. In this application, the random variable chosen was the peak ground acceleration a_G. The results showed that, for some lateral deflection limits and specified target reliabilities, there could be more than one solution M (Foschi and Li, 1999).

In this paper, a modified inverse reliability procedure will be proposed to consider the problem of multiple solutions. With this procedure, all the solutions can be obtained and the "best" can be chosen on the basis of sensitivity analysis. In order to introduce the method, the same pile foundation problem discussed by the authors in the previous paper is again used.

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PROPOSED METHOD

A general method was developed to approach the inverse reliability problem for either a single or multiple design variables by Li and Foschi (1998). In general, these are also regarded as random variables, with corresponding mean values and standard deviations. The problem may include the determination of both, or the means alone when the coefficient of variations are specified, or the standard deviations when the means are given. The detailed description of the method is given elsewhere (Li and Foschi, 1998). To facilitate this discussion, the following is a brief description for the case of a single design parameter.

Suppose that the limit-state function G in the Standard Normal, uncorrelated space is

$$G(\boldsymbol{u}) = g(\boldsymbol{x}, d) \tag{1}$$

where $\boldsymbol{u} = (\boldsymbol{u}_x, u_d)^T$ is the random, Standard Normal vector with components $\boldsymbol{u}_x$ and u_d corresponding, respectively, to the random variables $\boldsymbol{x}$ and the design variable d in the original, basic space. d is treated as an additional single random variable with a specified distribution (with mean value $\bar{d}$ and standard deviation σ).

For a target reliability index β, the inverse problem can be stated as:

$$\begin{aligned} &\text{Given } \beta, \\ &\text{Find: } \bar{d} \text{ (mean value of } d\text{) or } \sigma \text{ (standard deviation of } d\text{)} \\ &\text{Subject to: } \min(\boldsymbol{u}^T\boldsymbol{u}) = \beta^2 \text{ and } G(\boldsymbol{u}) = g(\boldsymbol{x}, d) = 0 \end{aligned} \tag{2}$$

The mapping of $\boldsymbol{u}_x$ and u_d into, respectively, $\boldsymbol{x}$ and d is achieved through the transformations

$$x_i = F_{x_i}^{-1}(\Phi(u_{xi})) \qquad i=1,2\ldots,n \tag{3}$$

$$d = F_d^{-1}(\Phi(u_d)) \tag{4}$$

where F_{x_i} and F_d are cumulative distribution functions for the variable x_i and d, respectively, and $\Phi(.)$ is the Standard Normal function.

Two basic formulae were derived to iterate the design parameter for a given target reliability index β (Li, and Foschi, 1998).

$$\boldsymbol{u} = \frac{-\beta\nabla_u G}{(\nabla_u G^T \nabla_u G)^{1/2}} \tag{5}$$

$$\bar{d} = \bar{d}_0 - \frac{G(\boldsymbol{u}_0, \bar{d}_0)}{\left(\frac{\partial G(\boldsymbol{u}_0, \bar{d})}{\partial \bar{d}}\right)\Big|_{\bar{d}_0}} \tag{6}$$

For an initial pair ($\bar{d}_0$, $\boldsymbol{u}_0$), and corresponding gradient $\nabla_u G(\boldsymbol{u}, \bar{d})\big|_{\boldsymbol{u}_0, \bar{d}_0}$, Eqs.(5) and (6) are used together to obtain both the vector $\boldsymbol{u}$ and design parameter $\bar{d}$ by means of a Newton-Raphson iteration.

As mentioned before, in earthquake engineering, the search for the value of the design parameter might offer difficulties, as several solutions may be available for the same design requirement. Also, the Newton-Raphson method may fail to find the solution due to the existence of the local extrema. Figure 1 gives an illustration of this problem. Assume that a curve representing the relationship between β and the design parameter d was built by repeating the forward reliability analysis.

At the target reliability level $\beta_T = \beta_1$ a unique d would be found, but at $\beta_T = \beta_2$, four values of d would be found, all of which meet the reliability requirement. For example, in dynamic analysis, different masses could result in the same response under a ground excitation due to the occurrence of resonance. The change of mass will change the natural frequency of the system, thus resulting in a variation of the mass at which resonance will occur. In such a case, the original direct inverse reliability method, which is based on the Newton-Raphson method, may fail to locate all the solutions or even fail to achieve convergence if the initial design parameter is not chosen carefully. Thus, there remain two problems: 1) how to locate all the solutions in the feasible domain and 2) how to choose a "best" solution among the multiple solutions.

By a combination of the Newton–Raphson method with the bisection method, all the solutions can be located, one by one, as long as one can isolate intervals for them. Let d_L be the lower bound and d_U the upper bound of a potential solution, and $G(d_L)$ the limit-state function at the lower bound and $G(d_U)$ the limit-state function at the upper bound. The modified procedure is described as follows:

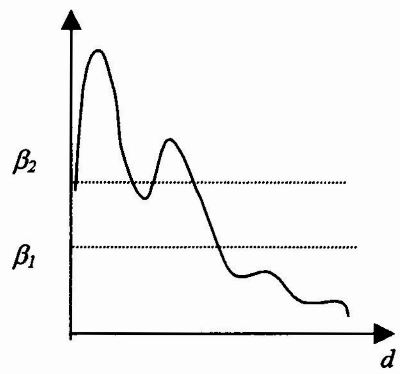

Figure 1: β vs. Design Parameter d

1) Assume that a lower bound d_L and a upper bound d_U of a potential solution are given, so that the product $G(d_L)G(d_U)$ is negative, indicating that a solution exists between d_L and d_U. Any value inside the interval can be chosen as the initial design parameter, d_0.

2) With an initial design point $\boldsymbol{u}$ and the d_0, the iterations start using the Newton-Raphson method and an upgraded design parameter d is obtained using Eq. (6). Should d be out of the given interval, instead of Newton-Raphson method, the bisection method is used to obtain the upgraded d by linear interpolation or taking average of the bounds, which ensures that the solution is in the interval. In the process of the iteration, the interval is narrowed using the newly obtained d. The solution can be found conditional on the $\boldsymbol{u}$ vector.

3) Conditional on the newly obtained design parameter d, the $\boldsymbol{u}$ vector is upgraded using Eq.(5). Then go back to step (2) to seek the new design parameter. The iterations are repeated until convergence is reached.

4) The next solution can be obtained by providing the corresponding new bounds.

The advantage of this hybrid method is that it will never fail to find the solution with a reasonable speed of convergence, since the bisection method will only be used when the solution is outside the bounds. It should also be noted that the drawback of this approach is that one has to provide the bounds ahead of the analysis, however as mentioned before, the initial deterministic analysis can provide such bounds approximately. Furthermore, in the implementation, due to the upgrading of the $\boldsymbol{u}$ vector, the inequality $G(D_L)G(D_U) < 0$ could not be always satisfied. If the produce $G(D_L)G(D_U)$ is positive, a slight adjustment for bounds will be used to change the bound which gives lower absolute value for G. After obtaining all the possible solutions, the most suitable solution, from a safety point of view, is the one which has the lowest sensitivity of the reliability index β with respect to the design parameter. The formula for calculating the sensitivity factor is given by:

$$\frac{d\beta}{dp} = \frac{\frac{dG}{dp}}{(\nabla_u G^T \nabla_u G)^{1/2}} \tag{7}$$

NUMERICAL EXAMPLE

This is a problem of designing a *single pile foundation* under earthquake loading, which demands a nonlinear dynamic analysis. A pile, as shown in Fig. 2, assumed elasto-plastic, is driven into a soil, which is modeled as a nonlinear foundation. The pile carries a mass M at its head and is excited by the free-field motion of the soil during an earthquake. The objective of this application is to calculate the mass M that can be supported while meeting a prescribed reliability in a serviceability limit-state. The pile is a 30.0m – long steel tube with an outside diameter of 356mm and a wall thickness of 10 mm.

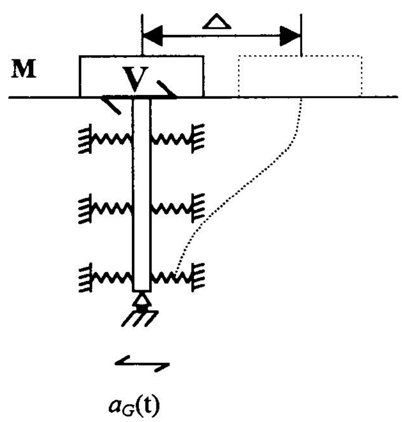

Figure 2: Pile in an Earthquake Excitation

Under the time-varying excitation, the relationship between the horizontal displacement Δ of the pile cap and the shear force V is typically in the form of a pinched hysteresis loop. This represents the nonlinear response resulting from the elasto-plastic properties of the pile and the nonlinear interaction between the surrounding soil and the pile, including gap formation.

Although many uncertainties are involved in this problem, associated with the earthquake as well as with the pile and the soil characteristics, it is assumed in this example that the random variables are the peak ground acceleration a_G, the yield stress σ_y of the pile and the soil relative density D_r. The earthquake excitation was considered as the historical event, the 1992 earthquake in Landers, California, Joshua Tree Station. The random variable a_G was assumed to be lognormal with mean value of 1.0 m/sec^2 and a coefficient of variation of 0.6, which is consistent with a design acceleration of 0.23g when a return rate of 0.05 was assumed for the earthquakes. Let R be a Normal distributed random variable associated with the uncertainty in the yield stress of the pile. The summary of the statistics of the random variables is given in Table 1. The design mass M is assumed Normal distributed with the coefficient of variation of 0.01. All variables are assumed uncorrelated.

The serviceability limit state function is defined as

$$G = D_0 - D_{max}(a_G, \sigma_y, M, D_r, ...) \quad (8)$$

where the maximum pile-cap displacement D_{max} is determined by the nonlinear dynamic analysis, as a function of two intervening random variables and other deterministic variables. D_0 is the maximum allowable pile-cap displacement, in this case, 0.10m. The reliability index against exceeding Δ_{max} during the earthquake event is a target $\beta = 2.5$, corresponding to an exceedence probability of approximately 6.2×10^{-3}.

Table 1: Statistics of the random variables

Variables	Mean Value	COV	Type
a_G	1.0m/sec^2	0.6	Lognormal
R, Associated with the yield stress (250.0Mpa)	1.0	0.1	Normal
M	?	0.01	Normal
D_r	75.0	0.2	Normal

From other work by the authors (Foschi and Li, 1999, Figure 4), when considering only one design parameter, the required mass is found at the intersection of the curve for $\beta = 2.5$ with a horizontal at the required limit of 0.10m, which gives three possible solutions for M. These are estimated to be $M = 43.12$, 48.92 and 55.62 Tonnes.

To upgrade these estimates using the inverse reliability technique, the influence of all uncertainties involved (yield stress, soil relative density and mass) is now considered in conjunction with the hybrid of the Newton-Raphson and the bisection method, as discussed before. To save computational effort, the response surface (an approximate representation of the maximum displacement) was first fitted in the neighbourhood of each of the estimated masses M. After few iterations, results were obtained as shown in Table 2, which gives all the (upgraded) possible design masses with corresponding sensitivities.

It is seen that the effect of considering all the uncertainties has, in this case, a very slight influence on the desired mass. The solution $M = 55.08$ Tonnes, with the lowest sensitivity, is chosen as the best solution.

Table 2: Summary of the Results ($\beta_{target} = 2.5$)

Cases	Design Mass (From initial deterministic analysis) (Tonnes)	Initial Mass Interval (Tonnes)	Design Mass (Tonnes)	Sensitivity
Solution 1	43.12	40.0-45.0	42.81	0.078
Solution 2	48.92	45.0-50.0	49.60	0.068
Solution 3	55.62	54.0-58.0	55.08	0.065

CONCLUSIONS

Combining Newton-Raphson's and a bisection method, an iteration technique was integrated with the inverse reliability method for the case when there are either 1) multiple solutions for a design parameter, or 2) difficulties in convergence due to local extrema. The approach was shown to be very useful in a problem involving nonlinear dynamic analysis in earthquake engineering.

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